Vertical motion

Vertical motion is any type of upwards or downwards motion that is constant. In a vertical motion problem, you may be asked about instantaneous velocity, and/or average velocity. To solve for instantaneous velocity we will need to take the derivative of our position function. The position function is $s(t)$ and the instantaneous velocity is $v(t)$ (which is $s'(t)$). To solve for average velocity we will need our general formula

$$\Delta v(t_1 t_2) = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

Remember, for instantaneous velocity we only require one time point but for average velocity we require two time points.

Example

A watermelon is dropped from the top of a building that's 28 meters high. The position function of the watermelon is given by

$$s(t) = -4t^2 + 28$$

where s is measured in meters, and t is measured in seconds.

1. What is the instantaneous velocity of the watermelon when $t = 4$?
2. What is the average velocity from $t = 2$ until $t = 5$?
3. When will the watermelon hit the ground?

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First we need to find the derivative of $s(t)$.

$$s'(t) = v(t) = -8t$$

To find velocity when $t = 4$, we'll plug $t = 4$ into $v(t)$.

$$v(4) = -8(4)$$

$$v(4) = -32$$

Our velocity at $t = 4$ is -32 m/s.

We'll use the formula for average velocity,

$$\Delta v(t_1, t_2) = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

and remember that $t_1 = 2$ and $t_2 = 5$.

$$\Delta v(t_1, t_2) = \frac{-4(5)^2 + 28 - \left[-4(2)^2 + 28\right]}{5 - 2}$$

$$\Delta v(t_1, t_2) = \frac{-100 + 28 + 16 - 28}{5 - 2}$$

$$\Delta v(t_1, t_2) = -28$$

Average velocity from $t = 2$ until $t = 5$ is -28 m/s.

When the watermelon hits the ground, $s(t) = 0$, because the position of the watermelon is at a height of 0 meters off the ground. Therefore, we need to set $s(t) = -4t^2 + 28$ equal to 0 and solve for t.

$$-4t^2 + 28 = 0$$

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$$-4t^2 = -28$$

$$t^2 = 7$$

$$t = \sqrt{7} \approx 2.65$$

The watermelon hits the ground at $t \approx 2.65$ s.

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